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METHODS OF HEAT COST ALLOCATION

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Abstract

Present article describes and compares different methods of calculations for heat cost allocation. It deals separately with the problems of the evaporating cost allocators for single-pipe heating systems.

Keywords: heating, single-pipe systems, cost allocator.

Introduction

There are several ways of allocating heating costs. The lump-sum payment is being replaced by the proportional division of costs on the basis of consumption. These methods are approximative. The cost allocating method to be applied is chosen by consumers. In present paper many versions are presented and compared, touching the further difficulties caused by evaporating cost allocators of single-pipe heating systems.

The 1st and 2nd paragraphs of the XLV/1991 law on consumption measuring states that the basis of heat consumption allocation can only be the heat consumption defined with a calibrated meter. This establishes the relation between the heat supplier and the consumers. The distribution of costs among the consumers, that is the allocation of costs is based on their own agreement. This agreement can be optional, so several methods can be developed. In the followings we will present some of the possible versions that may give a choice to consumers. We have also worked out a factual example to facilitate the choice.

We have chosen a panel building in Szigetszentmiklós as an example. The building is a five-stored residential house, with down-feed single-pipe heating with cross by pass sections. The rated temperature of heating water is 90/70 °C. We have disposed all of the design data necessary for calculations. Appendix I contains the results of calculations, including, besides the basic characteristics, the cost ratios of rooms of identical size and destination. The basis of comparison is always the room with the smallest cost. The example presents a typical corner- room.

1. Methods of Allocation

1.1. Lump-Sum Payment

This is a well known way of cost allocation. It consists of allocating the costs among the consumers on the simple proportional basis of heated air volume. Given an identical indoor height, the base of calculation can be the floor-space of the heated rooms. In this case rooms of the same size have the same costs. In formula

$$F_j = \frac{V_j}{\sum V_i} \cdot \sum F_i, \tag{1}$$

where F_j – is the cost of one consumer,
V_j – is the heated air volume of one consumer,
$\sum V_i$ – is the total heated air volume of all the consumers,
$\sum F_i$ – is the total heat cost.

In our example the ratio of costs is uniformly 1.00.

This method ignores completely the heat consumption. It does not motivate savings.

1.2. Allocation on the Basis of Design Heat Loss

The design heat loss of heated rooms has to be known for the application of this method. If the design documentation is not available, it can be estimated on the basis of radiator sizes: in the case of two-pipe heating systems, the nominal output of radiators can easily be calculated from the technological description and the design temperature of the heating water. In the case of single-pipe heating systems the estimation requires complex simulating and computerised processes.

For the following methods the heat demand of the rooms has always to be known.

If the heat requirements are known, the calculation is simple in this method as well:

$$F_j = \frac{Q_{rj}}{Q_{ri}} \sum F_i, \tag{2}$$

where Q_{rj} – is the heat requirement of rooms of each consumer,
Q_{ri} – is the total heat requirement of rooms of all the consumers.

If the nominal output of the radiators is available, as an extra variation the thermal output of pipes can also be included in calculations.

This method is consumption rational-like, but it does not motivate savings.

1.3. *Allocation with Product-scaled Appliances*

As practice has shown, in the case of some allocator appliances a proper scale (and appliance) has to be chosen for each heating body on the basis of heat requirement, so calculation is simple.

The scale has to be chosen according to the description of the manufacturer.

The costs can be divided directly according to the measured value, and the possibility of simple comparison is motivating for the consumer. In formula

$$F_j = \frac{h_j}{\sum h_i} \sum F_i, \tag{3}$$

where h_j – is the evaporated column of fluid on the scaled consumer allocator for each consumer, radiator or totally,
$\sum h_i$ – as before, the total of all the consumers.

This method is consumption rational, it motivates savings, but it does not take into consideration singular consumer characteristics. The choice of scale requires increased organisation, as it concerns the handling of hundreds of different scales.

The method of scale choice can be further refined, but in order to give a comprehensive view of calculations, from now on I am going to deal with allocators where the scale is constant according to the evaporated fluid. This is essentially based on the relation between the volume of evaporated fluid and the total value of the surface temperature of the radiator. This concerns two-sensor electrical cost allocators as well.

1.4. *Cost Allocation Exclusively on the Basis of Design Heat Loss Ratios*

According to the 3rd and 4th principle of the European Standard, in cost allocations we get a value in proportion of consumption of each radiator, if the temperature of radiators changes within the system in the same degree:

$$K_{i\text{tot}} = K_{Qi} \cdot K_{Ti} \cdot K_{Ci} \cdot T_{Ai} \cdot K_{Ei}. \tag{4}$$

The value in proportion of consumption of each radiator is the following:

$$H_i = h_i \cdot K_{i\text{tot}}. \tag{5}$$

The allocation of costs is based on to the following formula:

$$F_j = \frac{H_j}{\sum H_i} \cdot \sum F_i. \tag{6}$$

The method is relatively simple, but it does not take into consideration the thermal conditions of the building.

1.5. Allocation on the Basis of Constant Rated Base Part and Proportionate Part

The annual heat cost of a building (system) is determined in two ways. One is according to the accounts of the heat supplier and the building. Currently, in Hungary this consists of two parts: the base fee and the heat fee. The establishment of these fees belongs to the authority of local governments and it is defined by the features of the heat supply. Its ratios cannot form the ratios of allocation among the consumers (dwellers).

It is useful to divide the costs to base and proportionate parts in the case of heat allocators as well. Their proportion has to be defined with regards to the physical conditions of the building and the characteristics of cost allocators.

In practice their proportion is often taken for a constant value. The higher the base fee the closer we are to the lump-sum version, at the same time the weaker is motivation for savings.

In the example I have worked out the normally proposed 50–50% proportion. The basis of allocation is the value of heat requirement. The consumption proportionate part is:

$$H_j = h_j \cdot K_{i,\mathrm{tot}}. \tag{7}$$

The relation of the cost allocation is:

$$F_j = \frac{H_j}{\sum H_i} \cdot \frac{\sum F_i}{2} + \frac{K_{j,\mathrm{tot}}}{\sum K_{i,\mathrm{tot}}} \cdot \frac{\sum F_i}{2}, \tag{8}$$

where the first side is the consumption proportionate part and the second side of the sum is constant. The latter can be defined on the basis of the consumer's air volume or the proportion of floor-surface. The results of these can be found in parts 5a and 5b of Appendix I.

1.6. Allocation on the Basis of Physical Characteristics of the Building

The location of different rooms in the building influences significantly the heat consumption. Taking this into consideration thoroughly requires complicated calculations, so I have worked out a relative easily applicable method.

The process essentially consists of establishing the ratio of base costs and consumption proportionate costs for each room, taking into consideration the heat loss of pipes as well.

The calculation is based on the definition of specific rated heat loss. Compared to the above described methods, this one requires more data, as the air volume of the room is necessary, too.

$$q_j = \frac{Q_{nj}}{V_j}. \tag{9}$$

Besides, for two characteristic rooms the internal temperature has to be defined that it emerges when the radiators are turned off, but the pipes are emitting heat and in the neighbouring rooms the prescribed indoor air temperature is prevailing. In the followings I will call this temperature base temperature. For the calculation it is sufficient to apply the known relation of basic heat balance:

$$\sum A_i k_i (t_o - t_i) + Q_{\text{pipe}}, \tag{10}$$

where A_i – is the surface of the surrounding structures,
k_i – is the heat transmission factor of the surrounding structures,
t_o – is the base temperature to be calculated,
t_i – is the design temperature over the surrounding structures,
Q_{pipe} – is design thermal output of the pipes going through the room that are heating even with the radiators turned off.

It is sufficient to take into consideration the air filtration in the heat transmission factor of the doors and windows.

The selected rooms should be the ones with the highest and lowest specific heat loss, respectively, and without basic pipe. With the help of the base temperatures calculated according to the above relation and the specific heat loss we can get the base temperature of any room, using the following formula:

$$t_{oj} = \frac{t_{o2} - t_{o1}}{q_2 - q_1} \cdot (q_1 - q_j) + t_{o1}, \tag{11}$$

where the indexes mean:

j – for the room in question,
1 – for the room with the highest specific heat loss,
2 – for the room with the lowest specific heat loss.

If there is a basic pipe (in case of systems with vertical connection pipe), then their thermal output has to be subtracted of the heat loss when calculating the specific heat loss.

We divide into two parts the rated heat requirement of the rooms with the use of the base temperature:

$$Q_{rj} = Q_{raj} + Q_{rpj}, \tag{12}$$

where Q_{raj} – is the base heat loss of a room,
Q_{rpj} – is the heat loss of a room proportionate to the local regulation.

The division is made with the help of the base temperature characteristic to the room:

$$Q_{raj} = \frac{t_{oj} - t_k}{t_{bj} - t_k} \cdot Q_{hj}, \tag{13}$$

or

$$Q_{rpj} = Q_{rj} - Q_{raj}, \tag{14}$$

$$F_j = \frac{Q_{raj}}{\sum Q_{raj}} \cdot \sum F_{ai} + \frac{h_j \cdot K_{j,\text{tot}}}{\sum H_j \cdot K_{j,\text{tot}}} \sum F_{pi}, \tag{15}$$

where $K_{j,\mathrm{tot}}$ – is the relation mentioned above, but

$$K_{Qj} = Q_{rpj}\,. \tag{16}$$

$\sum F_a$ – is the total base cost,
$\sum F_P$ – is the total proportionate cost.

The allocation of the total costs can be done according to the base heat loss:

$$\sum F_{ai} = \frac{\sum Q_{raj}}{\sum Q_{ni}} \cdot \sum F_i \tag{17}$$

and

$$\sum F_{pi} = \sum F_i - \sum F_{ai}\,, \tag{18}$$

where $\sum F_{ai}$ – is the total heat cost.

1.7. *Allocation Based on the Modified Consumption Value*

The thermal output of the radiator in the function of the medium temperature is:

$$Q = a \cdot \Delta t_k^n\,, \tag{19}$$

where a – is constant,
n – is the exponent of the partial load. In the case of plate radiator it is, for example 1.33.

The relation of evaporation speed and excess temperature will be examined in details further on, which is:

$$\vartheta_f = d \cdot g0.387\,, \tag{20}$$

where d – is constant.

We can easily see that $\vartheta_f = \Delta t_k^n$ equals K_c, if we use the modifying factor to $n = 1.33$, for example if it equals 1.33, the relation between the thermal output and the display speed will be:

$$Q = b_g 0.515\,, \tag{21}$$

where b – is constant.

It can be ascertained that the relation between the thermal output and the evaporation speed is approximately of square root. So we get a more precise consumption value if the allocation is not linear, but using the $(h_j / h_j)^{0.5}$ ratio.

Among the presented examples, version number 6 is worked out according to the modified consumption value, the results can be seen in *Table 1*.

Naturally, several other methods can be developed, but in our opinion the most precise, still easily applicable method is the latter version, as it takes into consideration the location of rooms, the thermal output of pipes and it makes more precise the use of evaporating heat cost allocators.

2. Allocators in Single-pipe Heating Systems

In Hungary the single-pipe heating systems differ significantly from the western solutions. Their design raises many problems that cannot be definitively and satisfyingly solved with western methods. In the followings we will examine conditions that characterise Hungarian single-pipe heating systems.

As it is known, the cheapest way of heat cost allocation is the evaporating appliance. We will not describe its design in details, we will only briefly resume characteristics that are later necessary.

The appliance with the evaporating fluid in an ampoule is placed in the middle of the radiator's length, at 60–80% of its height. The difference between the temperature of the fluid and the medium temperature of the radiator (expressed by the so called c-value) is regulated by standards. This difference can be significant depending on the appliance and the radiator, so the mountings of the appliances have to be of the same construction type within the whole heating system. The c-value is defined with the following equation:

$$C = 1 - \frac{\Delta t_F}{\Delta t_m}, \tag{22}$$

where Δt_F – is the excess temperature of the fluid in relation to the air temperature,
Δt_m – is the excess temperature of the heating medium in relation to the prescribed air temperature of the radiator.

The 3. and 4. paragraphs of the European Standard determine that only evaporating heat cost allocators with c-values defined by measurings can be installed, if the measured value is not higher than 0.3, or for one quarter of the overall heating surface it can be 0.4 as a maximum, if the design temperature is not higher than 90 °C.

This factor depends, among others, on the temperature level as well, so if the temperature of heating is different in the design state, the c factor can have different values. This problem is valid for the electrical heat cost allocators, too.

Besides the deviation caused by heat transmission described above, there are other problems, as well, that I will present in details further below.

The emitted heat of the radiators is defined by the *Eq.* (19) equation:

$$Q = a \cdot \Delta t_k n, \tag{19}$$

where a – is the constant of radiator,
n – is the output exponent.

The heat consumption is its time integral:

$$Q = Q = \int_0^{\tau_0} Q \,\mathrm{d}\tau = a \cdot \int_0^{\tau_0} \Delta t_k \cdot \mathrm{d}\tau, \tag{23}$$

where τ_0 – is the end of the measuring period.

The evaporating function of the fluid in the heat cost allocators (according to BRUNATA Holding a/s) is:

$$E = \frac{\sqrt{\dfrac{Y}{0.0276634} + 1225} - 35}{5.91958 \cdot 10^{-8}} \cdot \frac{\sqrt{\dfrac{t_x + 273.16}{6311.8}}}{e^{t_x + 273.16}}, \tag{24}$$

where E – is the evaporating length in capillary (mm),
Y – is the number of days,
t_x – is the temperature (°C),
m_D – is the evaporating volume (g/h),
T_F – is the temperature (K).

The function from Ista GmbH is:

$$m_D = 0.02819 \cdot 10^{-5} \cdot (T_F)^{2.263} \ln \frac{1}{1 - 10^{8.516} - \dfrac{2632}{(T_F)^{0.00133}}}. \tag{25}$$

These functions in this form are very complicated to apply so we have looked for a new, approximate function we have defined the degree of evaporation for the same measuring period (190 days) and the most common range of temperature (20–80 °C). For the sake of easy comparison we have chosen the value belonging to 80 °C as the base of comparison. Between 50 °C and 80 °C we consider the approaching acceptable. We have defined the function with regression calculation.

$$g = 1.47 \cdot 10^{-6} (t - 20)^{2.585} \text{ gr/day}, \tag{26}$$

where t – is the temperature of the fluid (°C).

The functions can be seen in *Fig. 1*, as well.

Table 1. Allocation variations

Room $V = 48.6\ \text{m}^3$	Q_{heat} W	h scale	1 Lump-sum	2 Heat-loss	3 h_{ratio}	4 h_{ratio} 100% Q_{heat}	5a h_{ratio} 50% Q_{heat}	5b	6 h_{ratio} $Q_{base} = f(t_O)$	7 $(h_{hratio})^{0.512}$ $Q_{base} = f(t_O)$
IV. floor	1855	13	1.00	1.24	1.63	1.93	1.57	1.36	1.34	1.29
III. floor	1501	9	1.00	1.00	1.13	1.05	1.05	1.03	1.00	1.00
II. floor	1501	11	1.00	1.00	1.38	1.32	1.16	1.12	1.04	1.02
I. floor	1566	8	1.00	1.04	1.00	1.00	1.00	1.00	1.02	1.03
Ground floor	2430	12	1.00	1.62	1.50	2.33	1.97	1.52	1.79	1.70

$h = 10.2$

In the table we have illustrated the change of thermal output with regards to $\Delta t_k = t - 20$ °C. It can be seen from the results that the output exponent of

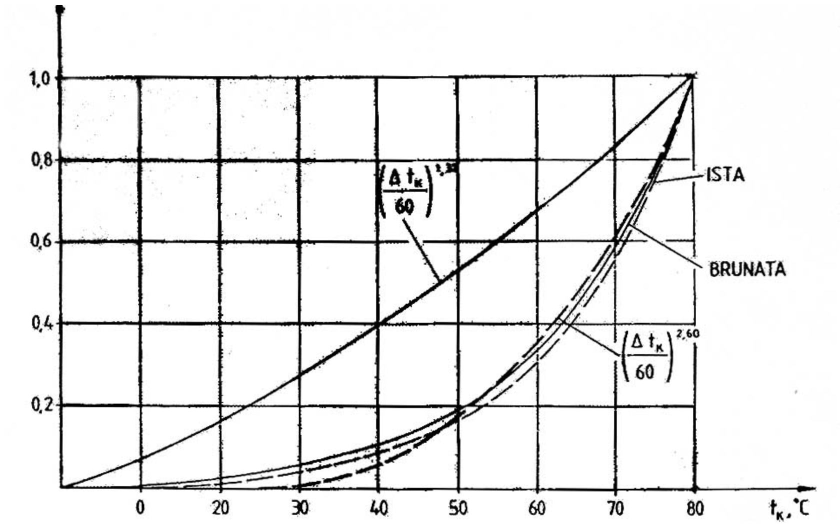

Fig. 1. Relative heat consumption and evaporation in function of the temperature during 190 days

the function is 1.33 and the evaporating function is 2.6. If we consider 80 °C as a base for the medium radiator temperature, and for example for a radiator of 60 °C medium temperature the ratio of output is 68% and the ratio of evaporation is 30% (or 31%). In other words, in a single-pipe heating installation, along the connecting pipe the temperature of the first radiator is 80 °C, and 60 °C of the last one, assuming a similar consumer attitude, the evaporation of the heat cost allocator at the last radiator is approximately a third of the evaporation at the first radiator. The read value has to be corrected.

In the European Standard there is an equation, which says with the original designations:

$$K_E = (K_{E,AL} - 1) \cdot 0.35 + 1 \tag{27}$$

and

$$K_{E,AL} = (v_{AN}/v_{HK}/\Delta t_{AN})^n, \tag{28}$$

where v_{HK} – is the evaporation speed at the zero fluid level of the scale on the examined radiator, in rated state,

v_{AN} – as above, but at the flow/return temperature of the system,

Δt_{HK} – is the logarithmic temperature difference of the examined radiator, in design state,

Δt_{AN} – as above, but counting with the flow/return temperature of the room,

n – is the exponent of the output function of the radiator.

Assigning the 80 °C and 60 °C used in the latest example to K_E we get 1.035. (For the medium temperature difference at 60 °C I calculated with 40 °C).

If we consider ideal the heat transmission of the cost allocator ($c = 0$), then displacing formula (26) in relation (19) we get

$$Q = b_g 0.515. \tag{21}$$

We have used $\Delta t_k = t - 20$ for the replacement. For the same period of time, at 80 °C and 60 °C, the ratio of heat consumption is 1.72. Comparing this value with the one defined in the European Standard we can see that the difference is enormous. So the development of a correction suitable for Hungarian conditions seems justified. For that purpose, measurings have to be carried out in apartment houses. Naturally, this error does not exist with electrical cost allocators.

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